ON MILSTEIN SCHEME FOR SOLVING GEOMETRIC BROWNIAN MOTION (GBM) EQUATION

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This study showed the application of explicit Euler and Milstein for solving a Geometric Brownian Motion (GBM). Using the Euler explicit scheme, it was observed that when the price of an asset at the initial time is positive, then the volatility of the asset is always positive; while if the price of the asset at the initial time is negative, the volatility of the asset is also negative. The GBM, discretizing in time T = 2, using the explicit Euler Scheme with constant volatility and drift shows the effect of random walk in stock prices. This shows that the degree of random walk is not entirely centered, and as such with timely variation of the drift and parameters can savage the stock price situation also the GBM through explicit Milstein scheme produced a chaotic process whose random walk is clustered with constant drift and volatility parameters. This suggests that the stock price situation will unlikely be savaged if the stock price market is sabotaged.

Key words: Stochastic Differential Equations (SDEs), strong and weak convergence, Milstein method.

INTRODUCTION

In deterministic differential equations, the consequence of random noise in the mathematical modelling of real-life situations is often ignored. It was noted that such equations only considered the mathematical framework of the system average (Cyganowski, 2002). However, when nonlinearities are involved in the model, it is required to present the complete performance of the model to access its behaviour. This is where the conception of stochastic differential equations is introduced. Stochastic differential equation describes the consequence of random noise within the physical systems. Stochastic differential equations have found many applications in science and technology such as Physics, Chemistry, structural mechanics and seismology, optical bistability and fatigue cracking, financial mathematics, mathematical biology, radio-astronomy, turbulent diffusion, etc. (Kloeden & Platen, 1992). A stochastic differential equation (SDE) is a differential equation in which one or more of the expressions are a stochastic process, ensuing in a result which is also a stochastic process. Typically, SDEs have a variable which represent random white noise considered as the derivative of Brownian motion or the Wiener process. However, other types of random behaviour are possible, such as jump process. Early work on SDEs was done to describe Brownian motion in Einstein's famous paper and at the same time by Smoluchowski (Li and Liu, 2017); though, one of the previous works associated to Brownian motion is credited to Bachelier (1901) in his thesis 'Theory of Speculation'. This work was followed by Langevin, and later Ito and Stratonovich placed SDEs on more Solid mathematical footing. Itô (1944) laid the foundation of a stochastic calculus known today as the Itô calculus. This represents the stochastic generalization of the classical differential calculus, which models various phenomena in continuous point in time such as the dynamics of stock prices, physical systems or the motions of a microscopic particle subject to random fluctuations. The corresponding stochastic differential equations (SDEs) generalize the ordinary deterministic differential equations (ODEs). In general, 1-dimensional Ito stochastic differential equation has the form

$$dX_t = \alpha(X_t)dt + \beta(X_t)dW_t,\ t \geq 0, \quad (1)$$

where $\alpha(X_t)$ is called the drift coefficient (which varies slowly), and $\beta(X_t)$ is the diffusion coefficient (a rapidly varying component). W_t is a Wiener process $W = \{W_t, t \geq 0\}$ that defines the randomness of the physical system, and it is often called the white noise. The subscript t in the white noise represents time-dependence.

The Wiener process is the simplest intrinsic noise term that adequately model Brownian motion. The integral form of Equation 1 is

$$X_t = X_0 + \int_0^t \alpha(X_s)ds + \int_0^t \beta(X_s)dW_s, t \geq 0 \quad (2)$$

The first integral in Equation 2 is a Volterra integral and the second integral is an Ito stochastic integral or an integral stochastic equation with respect to the Wiener process $W = \{W_t, t \geq 0\}$. More so, the second integral is not governed by the classical rule of calculus. This caused a period of stagnation in resolving this problem. However, it was not until the 1940s when Ito proposed his definition of the Ito integral that provided insight in resolving the second integral (Cootner, 2001).

Human life and human environment are inherently nonlinear and stochastic. Many model parameters that define any mathematical construction can only be estimated and also on the undeniable fact that many mathematical models are an approximation to reality. Thus, the numerical methods are required because it is difficult to solve SDEs analytically. Unfortunately, there was no explicit numerical method for stochastic differential equations, until the advent of digital super computers. Numerical methods such as the Euler's scheme, Milstein's scheme and Taylor's scheme, etc., were all implementable on digital computers.

The Taylor expansion is the bedrock for developing numerical approximations in deterministic calculus, so it is in stochastic numeric. Since the focus is on stochastic calculus; thus, a first order stochastic Taylor expansion has the form

$$g(X_t) = g(X_0) + L^0 g(X_0) \int_{t_0}^t ds + L^1 g(X_0) \int_{t_0}^t dW_s + \dots \quad (3)$$

where

$$L^0 = \frac{\partial}{\partial t} + \alpha(x)\frac{\partial}{\partial x} + \frac{1}{2}\beta(x)^2\frac{\partial^2}{\partial x^2},\ L^1 = \beta(x)\frac{\partial}{\partial x}. \quad (4)$$

and R is the remainder term. By applying the operator (Equation 4) repeatedly, higher order stochastic Taylor expansions were obtained.

Over the years, there are literatures that treated numerical approximation techniques for stochastic differential equations (SDEs). However, there is still a wide gap between the systematic theory of SDEs and its applications. In this study, this gap will be narrowed by exploring the numerical methods and their applications to Geometric Brownian Motion (GBM). All computational frameworks in this study are carried out with Maple 18 software.

Geometric Brownian motion (GBM)

The Geometric Brownian Motion (GBM) (also known as exponential Brownian motion) is most relevant in stock prices as it incorporates the fundamental of random walks in stock prices. A lot of researchers (Sengupta, 2004; Ladde and Wu, 2009; Wylomanska and Gajda 2012; Brewer et al., 2012; Abidin and Jaffar, 2014) over the years have used the GBM as a model in analyzing the degree of randomness in stock prices.

The GBM is a stochastic process $S(t)$, which is governed by the stochastic differential equation (SDE)

$$\frac{dS(t)}{S(t)} = \mu(t)\,dt + \sigma(t)\,dW(t) \quad 5)$$

where

$\mu = \mu(t)$ is the drift parameter

$\sigma = \sigma(t)$ is the volatility parameter

and

$W(t)$ is the standard Wiener process.

NUMERICAL METHODS

The explicit euler approximation

The Euler approximation (EP) is one of the elementary stochastic time discretization approximations of an Ito process (Reddy and Clinton, 2016). The scalar stochastic differential equation (SDE) of an Ito process is given as $W = \{W_t, t \geq 0\}$

$$dX_t = \alpha(t, X_t)dt + \beta(t, X_t)dW_t,\ t \geq 0, \tag{6}$$

with the initial condition

$$X_{t_0} = X_0.$$

Discretizing (6) in time in the interval $[t_0, T]$, we have

$$t_0 = \gamma_0 < \gamma_1 < \gamma_2 < \cdots < \gamma_n < \cdots < \gamma_N = T$$

which is a continuous Euler's approximation satisfying the scheme

$$X_{n+1} = X_n + \alpha(\gamma_n, X_n)(\gamma_{n+1} - \gamma_n) + \beta(\gamma_n, X_n)(W_{\gamma_{n+1}} - W_{\gamma_n}) \tag{7}$$

for $n = 0(1)(N-1)$, with the initial conditions

$$X_{t_0} = X_0,\ X_n = X(\gamma_n) \tag{8}$$

for all values of n at γ_n (discretization time). Now, let us write

$$\Delta_n = \gamma_{n+1} - \gamma_n \tag{9}$$

to denote the maximum nth time increment and call

$$\delta = \max_n \Delta_n \tag{10}$$

The maximum step in time increment

Let the equidistant time discretization be

$$\begin{cases} \gamma_{n+1} = \gamma_n + \delta \\ \quad or \\ \gamma_n = \gamma_n + n\delta \end{cases} \tag{11}$$

Thus, $\delta = \frac{(T-t_0)}{N}$ for N is large enough to ensure $\delta \in (0,1)$.

When $\beta(t, X_t) = 0$ in (6), the iterative scheme (7) reduces to an ordinary differential equation

$$dX_t = \alpha(t, X_t)dt \tag{12}$$

with the deterministic Euler scheme given as

$$X_{n+1} = X_n + \alpha(\gamma_n, X_n)(\gamma_{n+1} - \gamma_n),\ n = 0(1)(N-1) \tag{13}$$

The main difference between the stochastic iterative scheme (7) and the deterministic Euler iterative scheme is the term

$$\Delta W_n = W_{\gamma_{n+1}} - W_{\gamma_n},\ n = 0(1)(N-1) \tag{14}$$

where $W = \{W_t, t \geq 0\}$ is a Wiener process.

Chapter two acknowledges that

$E(W\Delta_n) = 0$, and $E[(W\Delta_n)^2] = t_{n+1} - t_n = \Delta_n$,

are Gaussian random variables (which are independent).

Using the above notations, the Euler discrete time approximation was rewritten as

$$X_{n+1} = X_n + \alpha\Delta_n + \beta\Delta W_n \tag{15}$$

An illustration of the simulation of the Euler's time discrete approximation is given below:

Let the Ito process $W = \{W_t, t \geq 0\}$ satisfies the linear SDE

$$dX_t = \alpha X_t dt + \beta X_t dW_t,\ t \geq 0 \tag{16}$$

with the initial condition $x_0 \in \mathbb{R}^1$.

Here, the drift ceefficient is

$$\alpha(t, X) = \alpha X$$

and diffusion coefficient is

$\beta(t,X) = \beta X$

Now to stimulate a sample path of the Euler scheme for (16), the initial approximation was taken from the initial value $X_{t_0} = X_0$, and proceed to generate respectively using

$$X_{n+1} = X_n + \alpha(\gamma_n, X_n)\Delta_n + \beta(\gamma_n, X_n)\Delta W_n, n = 0,1,2,3, \ldots, \quad (17)$$

with drift and diffusion coefficients well defined.

For instance, if

$\alpha(t,X) = \alpha e^{-X(t)} + \frac{1}{2}\beta^2 e^{-X(t)}$, $\beta(t,X) = \beta e^{-X(t)}$

with initial condition $X_{t_0} = X_0 = 1.0$.

Sample path can be generated for this stochastic process (in order to do this, numeric values must be assigned to α and β). To this effect, let $\alpha = 0.1$ and $\beta = 0.5$ be chosen arbitrary, the sample path with 100 time-steps is generated in Figure 1.

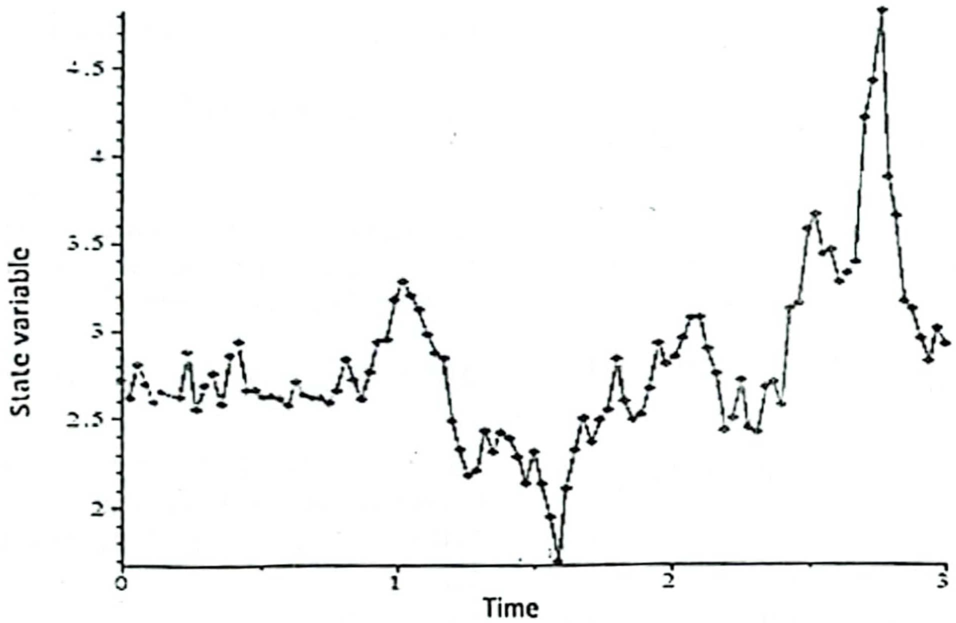

Figure 1. $PathPlot(e^{X(t)}, t = 0..3, timesteps = 100, replications = 1)$.

Strong convergence of the explicit Euler scheme

An approximating stochastic process X converges in strong sense if there is existing constant K and $\Delta_0 > 0$ with order $\tau \in [0, \infty)$ satisfying $E(|X_T - X_N|) \leq K\Delta_0^{\tau}$ with a maximum step size $\angle \in (0, \Delta_0)$ in any time discretization. When $\beta = 0$, the strong convergence of the stochastic Euler scheme becomes a mere deterministic Euler convergence scheme for the approximation of ordinary differential equation.

According to Milstein (1974), this strong convergence criterion is a measure of the absolute error at the final time interval T given as

$$\epsilon(\Delta_0) = E(|X_T - X_N|)$$

which can be deduced applying the Lyapunov inequality coupled with the root mean square error to be

$$\epsilon(\Delta_0) = E(|X_T - X_N|) \leq \sqrt{E(|X_T - X_N|^2)}$$

Platen (1981) and Kloeden (1992) argued that the order of strong convergence mechanism is much higher in the deterministic case than the stochastic case. In fact, the Euler approximation (7) has a strong convergence of order $\tau = 0.5$ in contrast with the Euler-approximation for deterministic ordinary differential equation, which has a strong order $\tau = 1.0$.

The theorem below is relevant in estimating the order of convergence of the stochastic explicit Euler scheme.

Theorem 1: Kloeden and Platen (1992)
Assuming that

$$E(|X_T|^2) < \infty \tag{18}$$

$$E(|X_T - X_N|^2)^{\frac{1}{2}} \leq K_1 \Delta_0^{\frac{1}{2}} \tag{19}$$

$$|\alpha(t,x) - \alpha(t,y)| + |\beta(t,x) - \beta(t,y)| \leq K_2|x-y| \tag{20}$$

$$|\alpha(t,x)| + |\beta(t,x)| \leq K_3(1 + |x|) \tag{21}$$

and

$$|\alpha(s,x) - \alpha(t,x)| + |\beta(s,x) - \beta(t,x)| \leq K_4(1 + |x|)|s-t|^{\frac{1}{2}} \tag{22}$$

for all $s,t \in [0,T]$ and $x,y \in R^2$, where $K_i, i = 1(2)4$ are independent of Δ_0. Then for the EP the estimate

$$E(|X_T - X_N|^2) \leq K_5 \Delta_0^{\frac{1}{2}} \tag{23}$$

Holds, where K_5 is independent of Δ_0. (Kloeden and Platen, 1992)

Weak convergence criterion of the stochastic explicit Euler scheme
An approximating stochastic process X converges in weak sense, when there exist a constant K and $\Delta_0 > 0$ (a positive constant) with order $\mu \in (0,\infty]$ and a polynomial f such that

$$|E(f(X_T)) - E(f(X_N))| \leq K\Delta_0^{\mu}$$

with a maximum step size $\Delta \in (0,\Delta_0)$ in any time discretization. When $\beta \equiv 0$, the weak convergence is the deterministic convergence criterion for the approximation of ordinary differential equation with $f(x) = x$.

Talay (1984) and Milstein (1978) stated that the explicit Euler-Maruyama approximation of a stochastic process has a weak order of 1.0, which is far superior to its strong convergence of order 0.5. Similarly, Platen and Mukulericius (1986) provide that the Euler-Maruyama converges with a weak order of 1.0 when the drift and diffusion coefficient of (7) are Holder continuous and Lipschitz continuous with fractional power.

The explicit Milstein scheme
Recall that the stochastic Taylors Formulae (STF) for the SDE (7) in the interval $t_0 \leq t \leq T$ is given as

$$g(t,X_t) = g(t,X_{t_0}) + d_1(t,X_{t_0})\int_{t_0}^{t} ds + d_2(t,X_{t_0})\int_{t_0}^{t} dW_s + d_3(t,X_{t_0})\int_{t_0}^{t}\int_{t_0}^{s_2} dW_{s_1}\, dW_{s_2} + R, \tag{24}$$

where,

$$d_1(t,x) = \alpha(t,x)g'(t,x) + \tfrac{1}{2}(\beta(t,x))^2 g''(t,x),$$

$$d_2(t,x) = \beta(t,x)g'(t,x),$$

$$d_3(t,x) = \beta(t,x)\{\beta(t,x)g''(t,x) + \beta'(t,x)g'(t,x)\}$$

Now, if we let $g(t,x) = x$ in the STF (24), we obtain

$$X_t = X_{t_0} + \alpha(X_{t_0})\int_{t_0}^{t} ds + \beta(X_{t_0})\int_{t_0}^{t} dW_s + \beta(X_{t_0})\beta'(X_{t_0})\int_{t_0}^{t}\int_{t_0}^{s_2} dW_{s_1}\, dW_{s_2} + R \tag{25}$$

which is a more general STF representation of the SDE (6) (Reddy and Clinton, 2016)
Now, if the last term of (25) is added to the numerical scheme in (3.21), the obtained Milstein scheme will be given as

$$X_{n+1} = X_n + \alpha\Delta_n + \beta\Delta W_n + \tfrac{1}{2}\beta\beta'\{(\Delta W_n)^2 - \Delta_n\},\ n = 0(1)(N-1) \tag{26}$$

Here, the additional term came from the double stochastic integral in (25), which can be computed using the Wiener increment ΔW_n

$$\int_{t_0}^{t}\int_{t_0}^{s_2} dW_{s_1}\, dW_{s_2} = \tfrac{1}{2}\{(\Delta W_n)^2 - \Delta_n\} \tag{27}$$

Generally, adding more stochastic integral terms in multiplicity to (24), a more strong Taylor approximation (STAs) was obtained. Such stochastic integral terms provides additional information about the discretized sample path. The theorem below gives conditions for Milstein scheme to ensure strong convergence of order $\tau = 1.0$.

Theorem 2: (Kloeden and Platen, 1992)
Assuming that

$$E(|X_T|^2) < \infty \quad (28)$$

$$E(|X_T - X_N|^2)^{\frac{1}{2}} \leq K_1 \Delta_0^{\frac{1}{2}} \quad (29)$$

$$|\underline{\alpha}(t,x) - \underline{\alpha}(t,y)| \leq K_2 |x-y| \quad (30)$$

$$|\beta^{j_1}(t,x) - \beta^{j_1}(t,y)| \leq K_2 |x-y|$$

$$|L^{j_1}\beta^{j_2}(t,x) - L^{j_1}\beta^{j_2}(t,y)| \leq K_2 |x-y|$$

$$|\underline{\alpha}(t,x)| + |L^{j}\underline{\alpha}(t,x)| \leq K_3(1 + |x|) \quad (31)$$

$$|\beta^{j_1}(t,x)| + |L^{j}\beta^{j_2}(t,x)| \leq K_3(1 + |x|)$$

$$|L^{j}L^{j_1}\beta^{j_2}(t,x)| \leq K_3(1 + |x|)$$

and

$$|\underline{\alpha}(s,x) - \underline{\alpha}(t,x)| \leq K_4(1 + |x|)|s-t|^{\frac{1}{2}} \quad (32)$$

$$|\beta^{j_1}(s,x) - \beta^{j_1}(t,x)| \leq K_4(1 + |x|)|s-t|^{\frac{1}{2}}$$

$$|L^{j_1}\beta^{j_2}(s,x) - L^{j_1}\beta^{j_2}(t,x)| \leq K_4(1 + |x|)|s-t|^{\frac{1}{2}}$$

for all $s,t \in [0,T]$ and $x,y \in R^d, j = 0,\dots,m$, and $j_1, j_2 = 1,\dots,m$, where $K_i, i = 1(2)4$ are independent of Δ_0. Then for the Milstein scheme the estimate

$$E(|X_T - X_N|^2) \leq K_5 \Delta_0^{\frac{1}{2}} \quad (33)$$

holds where K_5 is independent of Δ_0.
(See the proof in Kloeden and Platen, (1992)

RESULTS

Euler and Milstein schemes for the geometric Brownian motion

Implementing the Euler and Milstein schemes on the Geometric Brownian motion (5), the following obtained results were presented in graphs and tables (Table 1 and Figure 2). The maple generated GBM with timesteps 100 having 5 replications are the already existing drift and volatility; while the Euler generated GBM with timesteps 100 having 5 replications are the newly generated drift and volatility.

Table 1. Explicit Euler approximations for the GBM.

n	1	2	3	4
t	0.5	1	1.5	2
S_n	0.945	0.86	0.85	0.84

n = number, t = time and S_n = State Variable.

Table 2. Milstein approximations for the GBM.

n	1	2	3	4
t	0.5	1	1.5	2
S_n	1.2	1.46	1.65	0.68

n = number, t = time and S_n = State Variable.

DISCUSSION

The explicit Euler and Milstein schemes have been employed to solve the Geometric Brownian Motion equation. It was done successively with MAPLE 18 based on the following relations:

(i) The parameter s_0 defines the initial value of the underlying stochastic process, which is a real constant.
(ii) The parameter μ is the drift. In the simplest case of a constant drift μ is a real number. Time-dependent drift can be set either as an algebraic expression or as a Maple procedure. If μ is given as an algebraic expression, then the parameter t should be passed to specify which variable in μ should be used as a time variable.
(iii) The parameter σ is the volatility. Dissimilar to the drift parameter, the volatility can be constant or time-dependent. Unlike drift, volatility can involve other (one-dimensional) stochastic variables.
(iv) The scheme options specify the

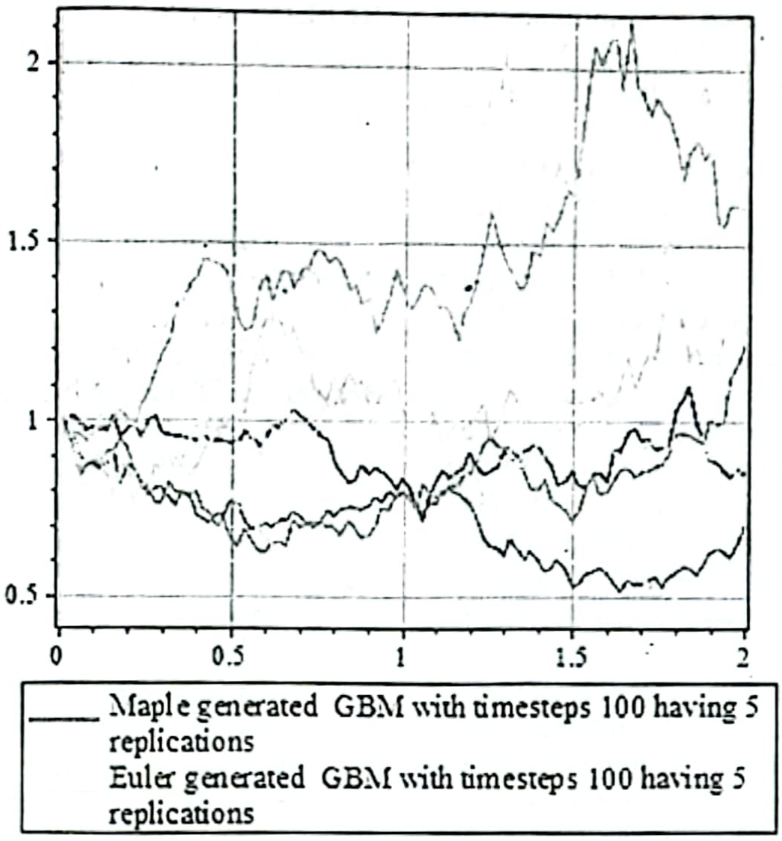

Figure 2. PathPlot for one-dimensional Brownian motion with constant drift and volatility using the Euler scheme.

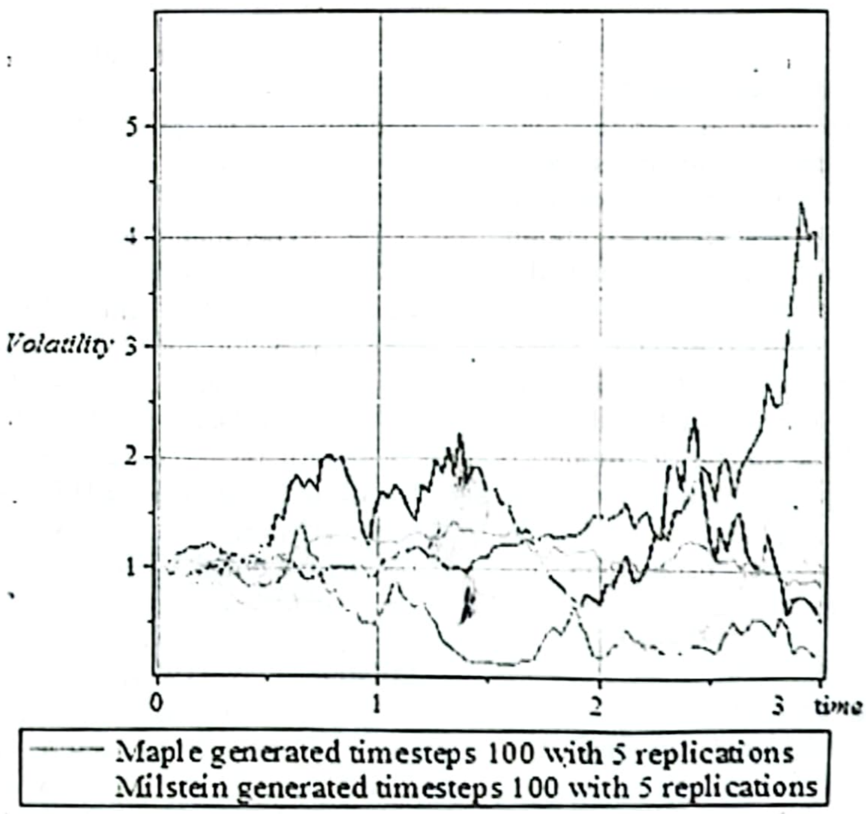

Figure 3. Path plot for one-dimensional Brownian motion with constant drift and volatility using the Milstein scheme.

discretization technique used for simulation of this process. The standard Euler and Milstein schemes were used. When a scheme is set to unbiased the transition density was used to simulate a value s (t + dt) given S (t). This scheme is appropriate in the case of a time-dependent drift and/or volatility.

(v) In the multi-dimensional case, the drift and the volatility parameter must be constant. The drift parameter must be specified as a Vector and the volatility parameter must be a symmetric matrix that defines the covariance between the individual components.

Now, discretizing a one dimensional GBM via explicit Euler scheme with constant drift and volatility at $T = 2$ shows the effect of random walk in stock prices. It shows that the degree of random walk is chaotic and as such with timely variation of the drift and parameters can savage the stock price situation. In like manner, the GBM through explicit Milstein scheme produced a nearly centered process whose random walk is clustered with constant drift and volatility parameters. This suggests that the stock price situation will be likely savaged if the stock price market is sabotaged.

Conclusion

Numerical methods have been used to solve a lot of complex mathematical formulations. This is because most analytic methods are so complex and difficult to implement. Stochastic differential equations (SDEs) are no exception. There are no precise analytic solvers for SDEs. Numerically, SDEs are often analysed through computer simulation. Thus, this paper solves the GBM equation by means of MAPLE 18 software, using both the explicit Euler and Milstein schemes. This stochastic model is mostly relevant in option pricing and stock price analysis. The results have shown that the drift and volatility are the parameters that determine system randomness and effects.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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